Edexcel Maths M1

Topic Questions from Papers

Vectors

7. Two ships P and Q are travelling at night with constant velocities. At midnight, P is at the point with position vector $(20\mathbf{i} + 10\mathbf{j})$ km relative to a fixed origin O. At the same time, Q is at the point with position vector $(14\mathbf{i} - 6\mathbf{j})$ km. Three hours later, P is at the point with position vector $(29\mathbf{i} + 34\mathbf{j})$ km. The ship Q travels with velocity $12\mathbf{j}$ km h^{-1}. At time t hours after midnight, the position vectors of P and Q are $\mathbf{p}$ km and $\mathbf{q}$ km respectively. Find

(a) the velocity of P, in terms of $\mathbf{i}$ and $\mathbf{j}$,

(2)

(b) expressions for $\mathbf{p}$ and $\mathbf{q}$, in terms of t, $\mathbf{i}$ and $\mathbf{j}$.

(4)

At time t hours after midnight, the distance between P and Q is d km.

(c) By finding an expression for $\overrightarrow{PQ}$, show that

$$d^{2} = 25t^{2} - 92t + 292.$$

(5)

Weather conditions are such that an observer on P can only see the lights on Q when the distance between P and Q is 15 km or less. Given that when $t = 1$, the lights on Q move into sight of the observer,

(d) find the time, to the nearest minute, at which the lights on Q move out of sight of the observer.

(5)

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Question 7 continued

Q7

(Total 16 marks)

TOTAL FOR PAPER: 75 MARKS

END

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8. [*In this question, the unit vectors* **i** *and* **j** *are horizontal vectors due east and north respectively.*]

At time $t = 0$, a football player kicks a ball from the point A with position vector $(2\mathbf{i} + \mathbf{j})$ m on a horizontal football field. The motion of the ball is modelled as that of a particle moving horizontally with constant velocity $(5\mathbf{i} + 8\mathbf{j})$ m s^{-1}. Find

(a) the speed of the ball,

(2)

(b) the position vector of the ball after t seconds.

(2)

The point B on the field has position vector $(10\mathbf{i} + 7\mathbf{j})$ m.

(c) Find the time when the ball is due north of B.

(2)

At time $t = 0$, another player starts running due north from B and moves with constant speed v m s^{-1}. Given that he intercepts the ball,

(d) find the value of v.

(6)

(e) State one physical factor, other than air resistance, which would be needed in a refinement of the model of the ball's motion to make the model more realistic.

(1)

Question 8 continued

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

END

Q8

N 2 0 9 1 1 A 0 2 0 2 0

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6. [*In this question the horizontal unit vectors* **i** *and* **j** *are due east and due north respectively*.]

A model boat A moves on a lake with constant velocity $(-\mathbf{i} + 6\mathbf{j})$ m s^{-1}. At time $t = 0$, A is at the point with position vector $(2\mathbf{i} - 10\mathbf{j})$ m. Find

(a) the speed of A,

(2)

(b) the direction in which A is moving, giving your answer as a bearing.

(3)

At time $t = 0$, a second boat B is at the point with position vector $(-26\mathbf{i} + 4\mathbf{j})$ m.

Given that the velocity of B is $(3\mathbf{i} + 4\mathbf{j})$ m s^{-1},

(c) show that A and B will collide at a point P and find the position vector of P.

(5)

Given instead that B has speed 8 m s^{-1} and moves in the direction of the vector $(3\mathbf{i} + 4\mathbf{j})$,

(d) find the distance of B from P when $t = 7$ s.

(6)

Question 6 continued

7. [*In this question the unit vectors* **i** *and* **j** *are due east and north respectively.*]

A ship S is moving with constant velocity $(-2.5\mathbf{i} + 6\mathbf{j})$ km h^{-1}. At time 1200, the position vector of S relative to a fixed origin O is $(16\mathbf{i} + 5\mathbf{j})$ km. Find

(a) the speed of S,

(2)

(b) the bearing on which S is moving.

(2)

The ship is heading directly towards a submerged rock R. A radar tracking station calculates that, if S continues on the same course with the same speed, it will hit R at the time 1500.

(c) Find the position vector of R.

(2)

The tracking station warns the ship's captain of the situation. The captain maintains S on its course with the same speed until the time is 1400. He then changes course so that S moves due north at a constant speed of 5 km h^{-1}. Assuming that S continues to move with this new constant velocity, find

(d) an expression for the position vector of the ship t hours after 1400,

(4)

(e) the time when S will be due east of R,

(2)

(f) the distance of S from R at the time 1600.

(3)

Question 7 continued

Q7

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

END

N 2 2 3 3 1 A 0 1 6 1 6

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3. A particle P of mass 2 kg is moving under the action of a constant force **F** newtons. When $t = 0$, P has velocity $(3\mathbf{i} + 2\mathbf{j})$ m s^{-1} and at time $t = 4$ s, P has velocity $(15\mathbf{i} - 4\mathbf{j})$ m s^{-1}. Find

(a) the acceleration of P in terms of **i** and **j**,

(2)

(b) the magnitude of **F**,

(4)

(c) the velocity of P at time $t = 6$ s.

(3)

Leave blank

7. A boat B is moving with constant velocity. At noon, B is at the point with position vector $(3\mathbf{i} - 4\mathbf{j})$ km with respect to a fixed origin O. At 1430 on the same day, B is at the point with position vector $(8\mathbf{i} + 11\mathbf{j})$ km.

(a) Find the velocity of B, giving your answer in the form $p\mathbf{i} + q\mathbf{j}$.

(3)

At time t hours after noon, the position vector of B is $\mathbf{b}$ km.

(b) Find, in terms of t, an expression for $\mathbf{b}$.

(3)

Another boat C is also moving with constant velocity. The position vector of C, $\mathbf{c}$ km, at time t hours after noon, is given by

$$\mathbf{c} = (-9\mathbf{i} + 20\mathbf{j}) + t(6\mathbf{i} + \lambda\mathbf{j}),$$

where λ is a constant. Given that C intercepts B,

(c) find the value of λ,

(5)

(d) show that, before C intercepts B, the boats are moving with the same speed.

(3)

N 2 6 1 1 4 A 0 1 8 2 0

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Question 7 continued

Q7

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

END

N 2 6 1 1 4 A 0 2 0 2 0

6. [*In this question, the unit vectors **i** and **j** are due east and due north respectively.*]

    A particle P is moving with constant velocity $(-5\mathbf{i} + 8\mathbf{j})$ m s^{-1}. Find

    (a) the speed of P,

    (2)

    (b) the direction of motion of P, giving your answer as a bearing.

    (3)

    At time $t = 0$, P is at the point A with position vector $(7\mathbf{i} - 10\mathbf{j})$ m relative to a fixed origin O. When $t = 3$ s, the velocity of P changes and it moves with velocity $(u\mathbf{i} + v\mathbf{j})$ m s^{-1}, where u and v are constants. After a further 4 s, it passes through O and continues to move with velocity $(u\mathbf{i} + v\mathbf{j})$ m s^{-1}.

    (c) Find the values of u and v.

    (5)

    (d) Find the total time taken for P to move from A to a position which is due south of A.

    (3)

Leave blank

Question 6 continued

3. A particle P of mass 0.4 kg moves under the action of a single constant force $\mathbf{F}$ newtons. The acceleration of P is $(6\mathbf{i} + 8\mathbf{j})$ m s^{-2}. Find

   (a) the angle between the acceleration and $\mathbf{i}$,

   (2)

   (b) the magnitude of $\mathbf{F}$.

   (3)

   At time t seconds the velocity of P is $\mathbf{v}$ m s^{-1}. Given that when $t = 0$, $\mathbf{v} = 9\mathbf{i} - 10\mathbf{j}$,

   (c) find the velocity of P when $t = 5$.

   (3)

1. A particle P moves with constant acceleration $(2\mathbf{i} - 5\mathbf{j})$ m s^{-2}. At time $t = 0$, P has speed u m s^{-1}. At time $t = 3$ s, P has velocity $(-6\mathbf{i} + \mathbf{j})$ m s^{-1}.

   Find the value of u.

   (5)

Leave blank

6. Two forces, $(4\mathbf{i} - 5\mathbf{j})$ N and $(p\mathbf{i} + q\mathbf{j})$ N, act on a particle P of mass m kg. The resultant of the two forces is $\mathbf{R}$. Given that $\mathbf{R}$ acts in a direction which is parallel to the vector $(\mathbf{i} - 2\mathbf{j})$,

   (a) find the angle between $\mathbf{R}$ and the vector $\mathbf{j}$,

   (3)

   (b) show that $2p + q + 3 = 0$.

   (4)

   Given also that $q = 1$ and that P moves with an acceleration of magnitude $8\sqrt{5}$ m s^{-2},

   (c) find the value of m.

   (7)

Leave blank

Question 6 continued

N 2 9 4 1 1 A 0 1 7 2 4

2. A particle is acted upon by two forces $\mathbf{F}_1$ and $\mathbf{F}_2$, given by

   $\mathbf{F}_1 = (\mathbf{i} - 3\mathbf{j})$ N,

   $\mathbf{F}_2 = (p\mathbf{i} + 2p\mathbf{j})$ N, where p is a positive constant.

   (a) Find the angle between $\mathbf{F}_2$ and $\mathbf{j}$.

   (2)

   The resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is $\mathbf{R}$. Given that $\mathbf{R}$ is parallel to $\mathbf{i}$,

   (b) find the value of p.

   (4)

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8.   [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively.*]

A hiker *H* is walking with constant velocity $(1.2\mathbf{i} - 0.9\mathbf{j})$ m s^{-1}.
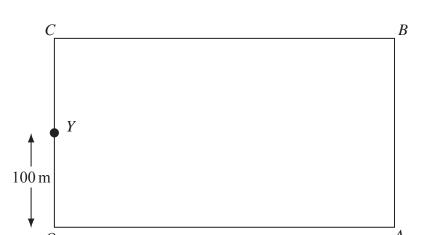
(a) Find the speed of *H*.

(2)

Figure 3

A horizontal field *OABC* is rectangular with *OA* due east and *OC* due north, as shown in Figure 3. At twelve noon hiker *H* is at the point *Y* with position vector 100 **j** m, relative to the fixed origin *O*.

(b) Write down the position vector of *H* at time *t* seconds after noon.

(2)

At noon, another hiker *K* is at the point with position vector $(9\mathbf{i} + 46\mathbf{j})$ m. Hiker *K* is moving with constant velocity $(0.75\mathbf{i} + 1.8\mathbf{j})$ m s^{-1}.

(c) Show that, at time *t* seconds after noon,

$$\overrightarrow{HK} = \left[(9 - 0.45t)\,\mathbf{i} + (2.7t - 54)\,\mathbf{j} \right] \text{ metres.}$$

(4)

Hence,

(d) show that the two hikers meet and find the position vector of the point where they meet.

(5)

N 3 4 2 7 5 A 0 2 0 2 4

Question 8 continued

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

END

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7. [*In this question*, **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin*.]

A ship S is moving along a straight line with constant velocity. At time t hours the position vector of S is **s** km. When $t = 0$, $\mathbf{s} = 9\mathbf{i} - 6\mathbf{j}$. When $t = 4$, $\mathbf{s} = 21\mathbf{i} + 10\mathbf{j}$. Find

(a) the speed of S,

(4)

(b) the direction in which S is moving, giving your answer as a bearing.

(2)

(c) Show that $\mathbf{s} = (3t + 9)\mathbf{i} + (4t - 6)\mathbf{j}$.

(2)

A lighthouse L is located at the point with position vector $(18\mathbf{i} + 6\mathbf{j})$ km. When $t = T$, the ship S is 10 km from L.

(d) Find the possible values of T.

(6)

M 3 5 1 0 4 A 0 2 0 2 4

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Question 7 continued

Q7

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

END

M 3 5 1 0 4 A 0 2 3 2 4

1. A particle P is moving with constant velocity $(-3\mathbf{i} + 2\mathbf{j})$ m s^{-1}. At time $t = 6$ s P is at the point with position vector $(-4\mathbf{i} - 7\mathbf{j})$ m. Find the distance of P from the origin at time $t = 2$ s.

(5)

4. A particle P of mass 2 kg is moving under the action of a constant force $\mathbf{F}$ newtons. The velocity of P is $(2\mathbf{i} - 5\mathbf{j})$ m s^{-1} at time $t = 0$, and $(7\mathbf{i} + 10\mathbf{j})$ m s^{-1} at time $t = 5$ s.

Find

(a) the speed of P at $t = 0$,

(2)

(b) the vector $\mathbf{F}$ in the form $a\mathbf{i} + b\mathbf{j}$,

(5)

(c) the value of t when P is moving parallel to $\mathbf{i}$.

(4)

Question 4 continued

N 3 5 4 0 7 A 0 9 2 4

7. [*In this question* **i** *and* **j** *are unit vectors due east and due north respectively. Position vectors are given relative to a fixed origin O.*]

Two ships P and Q are moving with constant velocities. Ship P moves with velocity $(2\mathbf{i} - 3\mathbf{j})$ km h^{-1} and ship Q moves with velocity $(3\mathbf{i} + 4\mathbf{j})$ km h^{-1}.

(a) Find, to the nearest degree, the bearing on which Q is moving.

(2)

At 2 pm, ship P is at the point with position vector $(\mathbf{i} + \mathbf{j})$ km and ship Q is at the point with position vector $(-2\mathbf{j})$ km.

At time t hours after 2 pm, the position vector of P is $\mathbf{p}$ km and the position vector of Q is $\mathbf{q}$ km.

(b) Write down expressions, in terms of t, for

    (i) $\mathbf{p}$,

    (ii) $\mathbf{q}$,

    (iii) $\overrightarrow{PQ}$.

(5)

(c) Find the time when

    (i) Q is due north of P,

    (ii) Q is north-west of P.

(4)

Leave
blank

Question 7 continued

24

Q7

TOTAL FOR PAPER: 75 MARKS

END

24

P 3 8 1 6 1 R R A 0 2 4 2 4

3. Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ acting on a particle P are given by

$$\mathbf{F}_1 = (7\mathbf{i} - 9\mathbf{j})\ \mathrm{N}$$
$$\mathbf{F}_2 = (5\mathbf{i} + 6\mathbf{j})\ \mathrm{N}$$
$$\mathbf{F}_3 = (p\mathbf{i} + q\mathbf{j})\ \mathrm{N}$$

where p and q are constants.

Given that P is in equilibrium,

(a) find the value of p and the value of q.

(3)

The force $\mathbf{F}_3$ is now removed. The resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is $\mathbf{R}$.
Find

(b) the magnitude of $\mathbf{R}$,

(2)

(c) the angle, to the nearest degree, that the direction of $\mathbf{R}$ makes with $\mathbf{j}$.

(3)

7. [*In this question, the unit vectors* **i** *and* **j** *are due east and due north respectively. Position vectors are relative to a fixed origin O*.]

A boat P is moving with constant velocity $(-4\mathbf{i}+8\mathbf{j})$ km h^{-1}.

(a) Calculate the speed of P.

(2)

When $t = 0$, the boat P has position vector $(2\mathbf{i}-8\mathbf{j})$ km. At time t hours, the position vector of P is **p** km.

(b) Write down **p** in terms of t.

(1)

A second boat Q is also moving with constant velocity. At time t hours, the position vector of Q is **q** km, where

$$\mathbf{q}=18\mathbf{i}+12\mathbf{j}-t(6\mathbf{i}+8\mathbf{j})$$

Find

(c) the value of t when P is due west of Q,

(3)

(d) the distance between P and Q when P is due west of Q.

(3)

Leave
blank

Question 7 continued

P 4 0 0 9 6 A 0 2 1 2 8

6. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.*]

A ship S is moving with constant velocity $(-12\mathbf{i} + 7.5\mathbf{j})$ km h^{-1}.

(a) Find the direction in which S is moving, giving your answer as a bearing.

(3)

At time t hours after noon, the position vector of S is **s** km. When $t = 0$, $\mathbf{s} = 40\mathbf{i} - 6\mathbf{j}$.

(b) Write down **s** in terms of t.

(2)

A fixed beacon B is at the point with position vector $(7\mathbf{i} + 12.5\mathbf{j})$ km.

(c) Find the distance of S from B when $t = 3$

(4)

(d) Find the distance of S from B when S is due north of B.

(4)

Leave blank

Question 6 continued

P 4 0 6 8 9 A 0 1 9 2 8

6. [*In this question,* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.*]

A ship sets sail at 9 am from a port P and moves with constant velocity. The position vector of P is $(4\mathbf{i} - 8\mathbf{j})$ km. At 9.30 am the ship is at the point with position vector $(\mathbf{i} - 4\mathbf{j})$ km.

(a) Find the speed of the ship in km h^{-1}.

(4)

(b) Show that the position vector **r** km of the ship, t hours after 9 am, is given by
$\mathbf{r} = (4 - 6t)\mathbf{i} + (8t - 8)\mathbf{j}$.

(2)

At 10 am, a passenger on the ship observes that a lighthouse L is due west of the ship. At 10.30 am, the passenger observes that L is now south-west of the ship.

(c) Find the position vector of L.

(5)

Question 6 continued

6. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively. Position vectors are given with respect to a fixed origin O.*]

A ship S is moving with constant velocity $(3\mathbf{i} + 3\mathbf{j})$ km h^{-1}. At time $t = 0$, the position vector of S is $(-4\mathbf{i} + 2\mathbf{j})$ km.

(a) Find the position vector of S at time t hours.

(2)

A ship T is moving with constant velocity $(-2\mathbf{i} + n\mathbf{j})$ km h^{-1}. At time $t = 0$, the position vector of T is $(6\mathbf{i} + \mathbf{j})$ km. The two ships meet at the point P.

(b) Find the value of n.

(5)

(c) Find the distance OP.

(4)

Leave blank

Question 6 continued

P 4 2 9 5 8 A 0 2 1 3 2

7. *[In this question, the horizontal unit vectors **i** and **j** are directed due east and due north respectively.]*

The velocity, **v** m s⁻¹, of a particle P at time t seconds is given by

$$\mathbf{v} = (1 - 2t)\mathbf{i} + (3t - 3)\mathbf{j}$$

(a) Find the speed of P when $t = 0$

(3)

(b) Find the bearing on which P is moving when $t = 2$

(2)

(c) Find the value of t when P is moving

    (i)   parallel to **j**,

    (ii) parallel to $(-\mathbf{i} - 3\mathbf{j})$.

(6)

Question 7 continued

P 4 1 8 2 8 A 0 2 3 2 8